ANALYSIS

This part of the site is devoted to analysis - the sort of thing that gives the struggling calculus student sweaty palms when they hear references to epsilon - delta proofs, uniform continuity etc. Analysis is often regarded as really hard but in my experience this is usually because it can be taught extremely poorly. There are in fact several very lucid communicators in the area. Elias Stein for instance won the Wolf Prize in part for the quality of his exposition of functional analysis concepts (his Princeton Series of Lectures in conjunction with Rami Shakarchi are wonderful books). David Bressoud has also produced some superb books aimed at making analysis much more understandable at a practical level and in an historical context.

The aim is to give detailed but comprehensible explanations of fundamental analytical techniques. Fourier Theory is the jewel in the crown when it comes to the application of analytical techniques and over time I will populate the site with material based around it, covering both the pure mathematics behind it but also the applications which are incredibly wide.

1. If you want to understand how to develop the binomial series for negative integral exponents read this article: The binomial series for negative integral exponents.pdf

https://gotohaggstrom.com/The binomial series for negative integral exponents.pdf

2. If you want to develop a deeper knowledge of some fundamental analytical techniques of Fourier theory why not try my paper titled: The good, the bad and the ugly of kernels: Why the Dirichlet kernel is not a good kernel". It derives its inspiration from Elias Stein and Rami Shakarchi's book "Fourier Analysis: An Introduction". Download my paper here

https://gotohaggstrom.com/Dirichlet kernel.pdf

3. Dirichlet's test for the convergence of series. In this short article I fully develop the test which is useful for oscillatory series. There is a derivation of the summation by parts formula. Click here to download the article: Dirichlet's test for convergence

https://gotohaggstrom.com/Dirichlet's test for convergence.pdf

4. Applying Riemannian integration theory to some practical examples. This is the mathematical

equivalent of building a pyramid with a trowel, but having done it once (an only once!) you will appreciate the power of the theory. The article can be accessed here: Riemann integration- some practical examples.pdf

https://gotohaggstrom.com/Riemann integration- some practical examples.pdf

5. If you want to brush up on some more advanced inequality techniques have a look at this paper which takes you through the solution process for some more difficult types of inequalities. You will need a knowledge of the Arithmetic Mean –Geometric Mean inequality and the Cauchy-Schwarz in equality. The Rearrangement Theorem and Chebyshev’s inequality are also covered. Download here: Advanced inequality manipulations.pdf

https://gotohaggstrom.com/Advanced inequality manipulations.pdf

6. When Schwarz proved his famous inequality he used an insight that enables a one line proof. To understand that proof click here: A one line proof of the Cauchy-Schwarz inequality.pdf

https://gotohaggstrom.com/A one line proof of the Cauchy-Schwarz inequality2.pdf

7. Proving the uniform continuity of sin x/x with some connections to Fourier transform theory. If you really want to get your hands dirty with epsilon-delta proofs go no further: Uniform continuity of sinc x.pdf

https://gotohaggstrom.com/Uniform continuity of sinc x.pdf

8. A short paper on logical manipulations for beginning analysis students: Basic logic for first year analysis students.pdf

https://gotohaggstrom.com/Basic logic for first year analysis students.pdf

9. I have substantially expanded the existing paper on the wave equation and energy conservation to cover the general case of d dimensions where d > 1.This involves a level of mathematical sophistication far greater than the one dimensional case dealt with initially. A fundamental part of the derivation involves differentiation under the integral sign, which requires a detailed discussion of the d dimensional Leibniz rule. The Reynolds Transport Equation is effectively this rule and there is a detailed discussion of Harley Flanders' reconciliation of mathematicians' and physicists' proofs of the Leibniz rule. You are warned – this is not for the faint hearted but I have put in all the detail so you should not get lost. Download the new paper here: The wave equation and energy conservation.pdf

https://gotohaggstrom.com/The wave equation and energy conservation.pdf

10. While it is possible to apply Fourier theory without knowing precisely why all the integrals converge, for those who worry about such things or actually have to demonstrate some understanding in an analysis exam, my detailed paper on "Basic Fourier Integrals" may help. It expands material covered in Elias Stein and Rami Shakarchi's Princeton Lectures on Fourier Theory. The concept of Schwartz space is developed in detail against the background of older approaches. Applications of Fourier theory are explained and the use of Fourier transforms to solve the Black-Scholes equation from finance is done in great detail. Download the paper here: Basic Fourier integrals.pdf

https://gotohaggstrom.com/Basic Fourier integrals.pdf

11. While soft sand running with Bondi Beach's surfing physicist Ruben Meerman of the ABC's "Catalyst" program we were discussing, among other things, why 1-1+1-…=1/2. My paper on Cesaro summability explains what is going on with such series and how mathematicians, like Mussolini redefining lateness so the trains ran on time (undoubtedly a myth but it sounds good), redefine what sums of such series are so that you get nice behaviour. The basics of Cesaro summability

https://gotohaggstrom.com/The basics of Cesaro summability.pdf

12. Laplace’s method of estimating the leading order behaviour of certain integrals is a powerful technique which can be used to prove Stirling’s formula among many other things. The proof of Laplace’s method is a problem in Polya’s and Szego’s famous book “Problems and Theorems in Analysis 1”. Their proof is rigorous but skips many fine details (which is not surprising given the level at which the book is pitched) which I have filled in in the download: Laplace’s method for integral asymptotics.pdf

https://gotohaggstrom.com/Laplace’s method for integral asymptotics.pdf

13. The Fejer kernel figures prominently in the theory of convergence of Fourier series and, unlike the Dirichlet kernel, it is well behaved. This good behaviour is explained by the Fejer kernel’s use of Cesaro sums. My detailed paper sets out all the relevant derivations (with multiple styles of proof for the most important results). If you are doing some serious Fourier analysis this will be of interest. To download click here: The nitty gritty of Fejer's Theorem.pdf

https://gotohaggstrom.com/The nitty gritty of Fejer's Theorem.pdf

14. Tutorial on the uniform continuity of the Fourier Transform - Parts 1 and 2.pdf

If you want to understand why, using classical analysis techniques, the Fourier transform of a function is uniformly continuous read this paper and watch the accompanying videos. The two part tutorial can be viewed here:

Part 1   https://www.youtube.com/watch?v=RG-dQMbGnMI&feature=youtu.be

Part 2 https://www.youtube.com/watch?v=iKWBYkPndZY&feature=youtu.be

or downloaded in lower definition (.wmv files) here:

Part 1

Fourier1.wmv

http://gotohaggstrom.com/Fourier1.wmv

Part 2

Fourier2.wmv

https://gotohaggstrom.com/Fourier2.wmv

The accompanying paper can be downloaded here

https://gotohaggstrom.com/Tutorial on the uniform continuity of the Fourier Transform - Parts 1 and 2.pdf

15. Chebyshev's inequality for L^p spaces can be proved in 2 lines (as Steven Krantz does in his book "A Panorama of Harmonic Analysis") but it is instructive to expand the steps. Download the paper:

Proof of Chebyshev's inequality in L^p spaces.pdf

https://gotohaggstrom.com/Proof of Chebyshev's inequality in L%5Ep spaces.pdf

16. An Australian high school maths teacher, Eddie Woo, who seems to have some cachet with students, did a video on the limit of x^x as x approaches 0. I long ago gave up on high school maths but the video was so superficial I just had to fill the gaping holes: Is there a rigorous high school limit proof.pdf

https://gotohaggstrom.com/Is there a rigorous high school limit proof.pdf

17. Eli Stein and Rami Shakarchi pose a meaty problem on Hermite functions in their book "Fourier Analysis – An Introduction". Although their focus is not that of a quantum physics textbook (they are functional analysts), the problem is of interest from a number of perspectives. A full solution is given and contains proofs of several intermediate steps.

Hermite functions - a solution to a Stein and Shakarchi problem.pdf

https://gotohaggstrom.com/Hermite functions - a solution to a Stein and Shakarchi problem.pdf

18. Forgotten how to do square roots in your head with calculus? squareroot.mov

https://gotohaggstrom.com/squareroot.mov

19. Euler was adept at using L'Hopital's rule multiple times to calculate various "indeterminate" limits eg of the form 0/0. The following article gives an example of what Euler did and how you can get the same result without use of L'Hopital's rule.

Multiple applications of L'Hopital's rule.pdf

https://gotohaggstrom.com/Multiple applications of L'Hopital's rule.pdf

20. It is a fundamental result of Fourier theory that the Fourier transform of a Gaussian is a Gaussian. But what is the Laplace transform of a Gaussian? So here is a demonstration of both cases: The Laplace transform of a Gaussian.pdf

https://gotohaggstrom.com/The Laplace transform of a GaussianV3.pdf